

電気基礎数学 演習問題 解答例

1. (4) $\int (\log_a x)^2 dx$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\begin{aligned} \int (\log_a x)^2 dx &= \frac{1}{(\ln a)^2} \int (\ln x)^2 dx \\ &= \frac{1}{(\ln a)^2} \{ x(\ln x)^2 - \int 2(\ln x) dx \} \\ &= \frac{1}{(\ln a)^2} \{ x(\ln x)^2 - 2x(\ln x) + 2x \} \end{aligned}$$

[別解]

$\ln x = t$ と置く。

$$dx = x dt = e^t dt$$

$$\int (\log_a x dx)^2 = \frac{1}{(\ln a)^2} \int t^2 e^t dt$$

$$\begin{aligned} \int t^2 e^t dt &= t^2 e^t - 2(\int t e^t dt) \\ &= t^2 e^t - 2t e^t + 2e^t \end{aligned}$$

$$\therefore \text{与式} = \frac{1}{(\ln a)^2} \{ x(\ln x)^2 - 2x(\ln x) + 2x \}$$

$$1. (6) \int \sqrt{a^2 - x^2} dx$$

$$x = a \cdot \sin(t) \quad \text{と置く}$$

$$dx = a \cdot \cos(t) dt$$

$$\begin{aligned} \int \sqrt{a^2 - x^2} dx &= \int a \sqrt{(1 - \sin^2 t)} \cdot a \cdot \cos(t) dt \\ &= \int a^2 \cos^2 t dt \end{aligned}$$

$$= \frac{a^2}{2} \int (1 + \cos(2t)) dt$$

$$= \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) = \frac{a^2}{2} (t + \sin(t)\cos(t))$$

$$= \frac{a^2}{2} \sin^{-1} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2}$$