

微積分 最低限基礎事項70問

<数列の極限>

<積分法>

(1) 収束,発散(振動)の定義をいえ.

(22) 陰関数 (例 $x^2 + xy - y^2 = 2$)

(42) 分岐点の積分の仕方

(2) limitの線形性とは.

(23) 媒介変数表示 $\begin{cases} x=f(t) \\ y=g(t) \end{cases}$

① $\int \frac{x^4+1}{x^2-1} dx$ ② $\int \frac{x^2}{(x+2)^2(x+1)} dx$

(3) 東法・西法と limit

(4) はさみ打ちの原理をいえ.

(24) 三角関数

(43) 部分積分法をいへよ.

(5) $\lim_{n \rightarrow \infty} x^n =$

- ① sin x ② cos x ③ tan x

(44) 次の積分をせよ.

(6) $a_{n+1} = f(a_n)$ で与えられる数列 $\{a_n\}$

- ④ cot x

① $\int_0^{\pi} x \cos x dx =$

ル作図法をいへよ.

(25) 指数関数 ① e^x ③ a^x

② $\int_1^e \log x dx =$

(7) (無限級数 $\sum_{k=1}^{\infty} a_k = S$ とは

(26) eの性質を4つかいて.

③ $\int_0^1 x e^x dx =$

(8) 等比級数 $\sum_{k=1}^{\infty} ar^{k-1} =$

(27) 対数関数 ① $\log|x|$ ② $\log_a x$ (45) 三角関数の積分の方法をいへ.

① $\int_0^{\pi} \sin^2 x dx$

(9) 級数の和の存在に関する定理

(28) 対数微分法とは.

② $\int_0^{\pi/2} \sin 3x \sin 5x dx$

をいへよ.

(49) $y = x^x \Rightarrow \frac{dy}{dx} =$

(46) 置換積分法をいへよ. 基本形

<関数の極限>

(29) n次冪関数とは.

(10) 右(左)極限とは.

(30) ライブニッツの公式をいへ.

的ものを3種類いへ.

(11) はさみ打ちの原理をいへ.

<微分法の応用> (47) テーニカルを置換の例をいへ.

(12) 三角関数の基本 limit 1つ

(31) $y=f(x)$ 上の微分可能な点 $x=a$

① 分母が $(ax+b)^n$ の分式

(13) 指数関数の基本 limit 2つ

における接線, 法線の方程式は.

② $\sqrt{ax+b}$ の式

(14) 対数関数の基本 limit 3つ.

(32) 平均値の定理をいへよ.

③ $\sqrt{a^2-x^2}$ の式

(15) 関数 $f(x)$ が $x=a$ で連続とは

(33) $f'(x)$ と極値の関係をいへ.

④ $(x+a)^n$ の式 (48)

(16) 関数 $f(x)$ が $x=a$ で微分可能

(34) $y=f(x)$ がある区間で下凸(上凸)凸凹

⑤ $\tan \frac{x}{2} = t$ とおく置換 $\int \frac{dx}{\cos x}$

とはいかなることか.

(35) $f'(x)$ と凸の関係をいへ.

⑥ $f(e^x) \dots e^x$ と π の関数

<微分法>

(36) $y=f(x)$ の変曲点とは.

(49) \int (多項式) $e^{ax} dx$ の形の積分

微分法の公式をいへよ.

(37) $y=f(x)$ のグラフをかく手順をいへ.

はさみ打ち.

(17) ベーネズ $y=x^\alpha$ (α は実数)

(38) 整方程式 $f(x)=0$ の根 $x=d$

(49) $\int_0^{\pi} \sin(px) \sin(qx) dx =$

(18) 積 $y=uv$

と f の必要十分条件をいへ.

(50) Wallisの公式をいへよ

(19) 商 $y=\frac{u}{v}$

(39) σ : 1次連立式をいへ.

(51) $\int_a^b (x-\alpha)^p (x-\beta)^q dx =$

(20) 合成 $\begin{cases} y=f(z) \\ z=g(x) \end{cases} \Leftrightarrow y=f(g(x))$

(40) 2次連立式をいへ.

(52) ニュートン-ライプニッツの公式をいへよ.

(21) 逆関数 $y=f(x) \Leftrightarrow x=g(y)$

(41) テイラー展開をいへ.

毎日10分の努力が 合格への近道

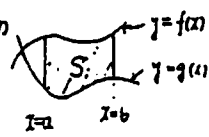
〈積分法的应用〉

- (51) 積分の平均値の定理をいえ。
- (52) 近似和の $\lim_{n \rightarrow \infty} (= \text{定積分})$ について説明せよ。
- (53) $\Delta y = f(x) \Delta x$ の誤差は Δx より高次の微小量とはどういうことか。

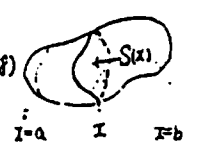
- (65) 点 $P(x, y)$ が平面運動をするとき、速度 (ベクトル) \vec{v} , 加速度 (ベクトル) \vec{a} は
 - (66) 点 P の時刻 t における位置を $P(t)$ とする ($t=1, 2$) とし、変位 (ベクトル), 道のり S は
- 〈微分方程式〉

(56) $dy = f(x) dx \Rightarrow \square$

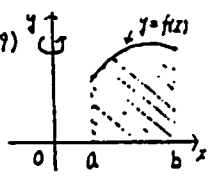
- (67) 変数分離形の1階微分方程式は、 $F(x, y)$ の解法をいえ。

(57)  左図の斜線部
面積 $S =$

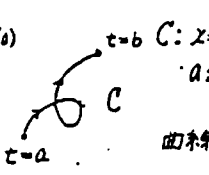
(例) $y' + y^2 = 0, y(0) = 1$ を解け。

(58)  左図の立体的
体積 $V =$

(68) (線形同次微分方程式)
 $y' + p(x)y = 0$ の一般解は

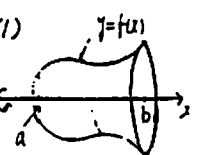
(59)  平面のまわりに
回転してできる
立体的の体積 $V =$

(69) (線形非同次微分方程式)
 $y' + p(x)y = g(x)$ の解法は

(60)  $t=b$ $C: x=f(t), y=g(t)$
 $a \leq t \leq b$
曲線 C の長さ $L =$

(例) $y' - 2y = e^x, y(0) = 1$ を解け。

(70) 2階微分方程式 (y と y' のみ含む)

(61)  x 軸のまわりに回転
してできる立体の
側面積 $S =$

$F(x, y, y') = 0$ の解法は。

(例) $y' + (y')^2 = 0$ を解け。

(71) (2階線形同次微分方程式)

(62) $S_n = \sum_{k=0}^{n-1} \frac{1}{n} f\left(\frac{k}{n}\right) n \Delta x, \lim_{n \rightarrow \infty} S_n =$

$y'' + p(x)y' + q(x)y = 0$ の解法
用いる定理をいへよ。

(72) $y' + ay = c (a \neq 0)$ の解法は。

(例) $S_n = \frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n-1}$
ただし、 $\lim_{n \rightarrow \infty} S_n$ を求めよ。

(73) $y'' + 2ay' + by = 0$ の解法は。

(例) $y'' + y' - 6y = 0$ を解け。

(74) (2階線形非同次微分方程式)

(63) シンチの各形公式をいへ。
〈点の運動と微積〉

$y'' + p(x)y' + q(x)y = r(x)$ の解法

用いる定理をいへよ。

(64) 点 $P(x)$ が直線運動をするとき、
速度 v , 加速度 a は

(1) $\lim_{n \rightarrow \infty} a_n = \alpha \Leftrightarrow \forall \epsilon > 0 \exists N \in \mathbb{N}$
 かつ $\forall n > N$ a_n は α と ϵ の間にあり
 かつ $\forall \epsilon > 0$, $\{a_n\}$ は α と近づく
 という。(発散 \Leftrightarrow 取れない)

(11) $\left. \begin{array}{l} x \rightarrow a \text{ 近く } \\ f(x) = g(x) = h(x) \\ \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = b \end{array} \right\} \Rightarrow \lim_{x \rightarrow a} g(x) = b$

(2) $\lim_{n \rightarrow \infty} (\alpha a_n + \beta b_n)$
 $= \alpha \lim_{n \rightarrow \infty} a_n + \beta \lim_{n \rightarrow \infty} b_n$

(12) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

(3) $\lim_{n \rightarrow \infty} (a_n b_n) = (\lim_{n \rightarrow \infty} a_n) (\lim_{n \rightarrow \infty} b_n)$
 $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n}$

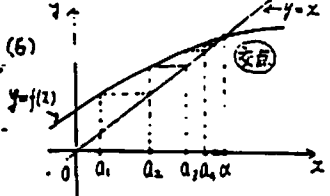
(13) ① $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$
 ② $\lim_{x \rightarrow \infty} \frac{x^\alpha}{e^x} = 0$ (α : 定数)

(4) $\left. \begin{array}{l} a_n \leq x_n \leq b_n \\ \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} b_n = \alpha \end{array} \right\} \Rightarrow \lim_{n \rightarrow \infty} x_n = \alpha$
 両側が α に近づくから、 x_n も α に近づく。

(14) ① $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$
 ② $\lim_{x \rightarrow 0} x^\alpha \log x = 0$ (α : 定数)
 ③ $\lim_{x \rightarrow \infty} \frac{\log x}{x^\alpha} = 0$

(5) $\lim_{n \rightarrow \infty} x^n = \begin{cases} +\infty & (x > 1) \\ 1 & (x = 1) \\ 0 & (|x| < 1) \end{cases}$
 振動 ($x = -1$)

(15) $\lim_{x \rightarrow a} f(x) = f(a)$
 (16) $f(a)$ が存在する。
 (17) $(x^\alpha)' = \alpha x^{\alpha-1}$



$\lim_{n \rightarrow \infty} a_n = \alpha$ (1. 存在するから)

(7) $\lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = S$ のとき、級数 $\sum_{k=1}^{\infty} a_k$ は収束し、その和は S である。

(8) $\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$ ($|r| < 1$)

(9) $\sum_{k=1}^{\infty} a_n$ が存在 $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$
 (証明) a_n が 0 に収束しないならば、級数の和は存在しない。

(10) $\lim_{x \rightarrow a} f(x) = b \Leftrightarrow \exists \delta > 0 \forall \epsilon > 0$
 δ といふ値をとりながら (右側から) a に近づくと $f(x)$ は b に近づくとする。

$\lim_{x \rightarrow 0} f(x) = b \Leftrightarrow$ (左側から)

(26) ① $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$
 ② $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$
 ③ $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

④ $e = \sum_{k=0}^{\infty} \frac{1}{k!}$

(27) ① $(\log x)' = \frac{1}{x}$
 ② $(\log_a x)' = \frac{1}{x \log a}$

(28) (a) $y = x^x$ $\log y = x \log x$
 $\frac{1}{y} \frac{dy}{dx} = \log x + x \cdot \frac{1}{x}$
 $\therefore \frac{dy}{dx} = y (\log x + 1)$
 $\therefore \frac{dy}{dx} = x^x (\log x + 1)$

(29) $f^{(n)}(x)$ は $\frac{d^n}{dx^n} f(x)$
 $f(x)$ が n 階微分可能なとき

(30) $(fg)^{(n)} = \sum_{r=0}^n nCr f^{(n-r)} g^{(r)}$
 (31) $y - f(a) = f'(a)(x - a)$
 $\therefore -f'(a)(y - f(a)) = (x - a)$

(32) $f(x)$ は (a, b) で増加可能
 (a, b) で連続
 $\Rightarrow \frac{f(b) - f(a)}{b - a} = f'(c), a < c < b$
 c が存在する。

(33) $f(a) = 0$ のとき $f'(a) > 0 \Rightarrow f(a)$ は極小値
 $f'(a) = 0$ のとき $f'(a) < 0 \Rightarrow f(a)$ は極大値

(34) $f(x)$ が下向き $\Leftrightarrow f'(x) < 0$ (増減)
 $f(x)$ が上向き $\Leftrightarrow f'(x) > 0$ (増減)

(35) $f(x) > 0 \Rightarrow f(x)$ は下向き
 $f(x) < 0 \Rightarrow f(x)$ は上向き

(36) 変曲点とは、その左右で凹凸が異なる点。

(37) $f(a) = 0$ (0 $\leq k < n-1$), $f^{(n)}(a) \neq 0$
 (38) $R = 0$ のとき
 (39) $f(a+h) = f(a) + h f'(a)$
 (40) $\frac{0}{0}, \frac{\infty}{\infty}$ の不定形

(41) $f(a+h) = f(a) + h f'(a) + \frac{1}{2} h^2 f''(a)$
 $\lim_{h \rightarrow 0} \frac{f(h)}{g(h)} = \lim_{h \rightarrow 0} \frac{f'(h)}{g'(h)}$

(42) (a) $\frac{x+1}{x^2-1} = x + \frac{x+1}{x^2-1}$
 $\frac{x+1}{x^2-1} = \frac{x+1}{(x-1)(x+1)}$
 $\frac{x+1}{x^2-1} = \frac{1}{x-1} + \frac{1}{x+1}$
 $\therefore a, b, c$ は存在する。

(43) $\int_0^b u'v dx = [uv]_a^b - \int_a^b uv' dx$
 ③ 三角関数は積分可能
 対数は対数微分可能

(44) $\int_0^{\pi} x \cos x dx$
 $= [x \sin x]_0^{\pi} - \int_0^{\pi} \sin x dx$
 $= [x \sin x]_0^{\pi} + [\cos x]_0^{\pi}$
 $= \pi \cdot 0 + (1 - 1) = 0$

(45) $\int_0^{\pi} \log x dx$
 $= [x \log x]_0^{\pi} - \int_0^{\pi} x \cdot \frac{1}{x} dx$
 $= \pi \log \pi - [x]_0^{\pi} = \pi \log \pi - \pi$

(46) $\int_0^1 x e^x dx$
 $= [x e^x]_0^1 - \int_0^1 e^x dx$
 $= 1 \cdot e - [e^x]_0^1 = e - (e - 1) = 1$

(47) $\int_0^1 x e^x dx$
 $= [x e^x]_0^1 - \int_0^1 e^x dx$
 $= 1 \cdot e - [e^x]_0^1 = e - (e - 1) = 1$

(48) $\int_0^{\pi} \sin^2 x dx$
 $= \int_0^{\pi} \frac{1 - \cos 2x}{2} dx$
 $= \frac{1}{2} [x - \frac{\sin 2x}{2}]_0^{\pi}$
 $= \frac{1}{2} \pi$

① 定数項と $\sqrt{\quad}$ (0 $\neq 0, \sqrt{\quad}$ の中)
 ② 対称性 (奇関数、偶関数)
 ③ y' を求め、増減表を書く (極値)
 (7) ① 不連続点に注意
 (1) $\lim_{x \rightarrow \infty} y, \lim_{x \rightarrow 0} y \rightarrow$ 漸近線 $x=0$
 (2) $\lim_{x \rightarrow \infty} \frac{y}{x} = m, \lim_{x \rightarrow \infty} (y - mx) = n$
 漸近線 $y = mx + n$ である。
 ④ y' を求め、凹凸を示す (変曲点)

<積分法の応用>

(3) $\int_0^{\frac{\pi}{2}} \sin 2x \sin x dx$
 $= \int_0^{\frac{\pi}{2}} \frac{1}{2} (\cos x - \cos 3x) dx$
 $= \frac{1}{2} \left[\frac{\sin x}{1} - \frac{\sin 3x}{3} \right]_0^{\frac{\pi}{2}} = 0$

(40) $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} f(x_k) (x_{k+1} - x_k)$
 (41) 区間 $[a, b]$ を n 等分する
 $\Delta x = \frac{b-a}{n}$

(42) $\frac{dy}{dx} = p(x)q(y)$ $\left[\frac{dy}{q(y)} = p(x) dx \right]$
 $\int \frac{dy}{q(y)} = \int p(x) dx$
 (43) $y = Ce^{-px}$

(44) $\varphi(x) = c \cdot x^a \cdot x^b$
 $\varphi'(x) dx = dt, \frac{x}{a} \rightarrow \frac{b}{a}$

(45) $\lim_{x \rightarrow 0} \frac{4x - f(x) \Delta x}{\Delta x} = 0$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = f(x) \Rightarrow \frac{dy}{dx} = f(x)$
 $\therefore [dy = f(x) dx]$

(46) $P(x) = \int p(x) dx, C$ は任意定数
 (47) 同次方程式 $y' + p(x)y = 0$
 \downarrow 変数分離 $u = e^{-\int p(x) dx}$

(7) $\frac{\varphi(x)}{\psi(x)}, [\varphi(x)]^m \varphi(x)$ の形
 (8) $f(\sin x) \cos x, f(\cos x) \sin x$
 (9) $f(\log x) \frac{1}{x}$ の形

(48) $[y(b) - y(a) = \int_a^b f(x) dx]$
 (49) $S = \int_a^b |f(x) - g(x)| dx$
 (50) $V = \int_a^b S(x) dx$

(48) 同次方程式 $y' + p(x)y = 0$
 \downarrow 変数分離 $u = e^{-\int p(x) dx}$

(47) ① $ax + b = c$ とおく
 ② $\sqrt{ax+b} = c$ とおく
 ③ $x = a \cos \theta$ ($0 \leq \theta \leq \pi$) とおく
 ④ $x = a \tan \theta$ ($-\frac{\pi}{2} < \theta < \frac{\pi}{2}$) とおく
 ⑤ $\cos x = \frac{t-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$
 $dx \rightarrow \frac{2dt}{1+t^2}$
 ⑥ $e^x = t$ とおく

(51) $V = \int_a^b x f(x) dx$
 (52) $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
 (53) $S = 2x \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

(49) $y' - 2y = 0$ の解 $u = e^{2x}$ とおく
 $y = x e^{2x}$ とおく。導関数 $y' = 2x e^{2x} + e^{2x}$
 $x e^{2x} + 2x e^{2x} = y - e^{2x}$
 $\therefore x' = e^{-2x}$ より $\int dx = \int e^{-2x} dx$
 $\therefore x = -e^{-2x} + C$
 (50) $y = (-e^{-x} + C) e^{2x}$

(47) ① $\sqrt{ax+b} = c$ とおく
 ② $x = a \cos \theta$ ($0 \leq \theta \leq \pi$) とおく
 ③ $x = a \tan \theta$ ($-\frac{\pi}{2} < \theta < \frac{\pi}{2}$) とおく
 ④ $\cos x = \frac{t-t^2}{1+t^2}, \sin x = \frac{2t}{1+t^2}$
 $dx \rightarrow \frac{2dt}{1+t^2}$
 ⑥ $e^x = t$ とおく

(54) $\lim_{n \rightarrow \infty} S_n = \int_a^b f(x) dx$
 (55) $f(x) = \frac{1}{n^2}$ とおく
 $S_n = \sum_{k=0}^{n-1} \frac{1}{n^2} = \sum_{k=0}^{n-1} \frac{1}{n} f\left(\frac{k}{n}\right)$

(51) $y' = u$ とおく \rightarrow 1階微分方程式
 (52) $y' = u$ とおく $u' + u^2 = 0$
 $\therefore \frac{du}{dx} = -u^2 \therefore \int \frac{du}{u^2} = \int dx$
 $\therefore \frac{1}{u} = x + C$
 (53) $y' = \frac{1}{x+c} \therefore \frac{dy}{dx} = \frac{1}{x+c}$

(48) $\int p(x) e^{ax} dx$ の形
 $\int p(x) e^{ax} dx$
 (49) $\int_0^x \sin(px) \sin(qx) dx = \begin{cases} \frac{x}{2} (p+q) & (p=q) \\ \frac{x}{2} (p-q) & (p \neq q) \end{cases}$

(56) $\int_a^b f(x) dx = S_n = \frac{h}{2} (y_0 + y_n) + h(y_1 + \dots + y_{n-1})$
 $h = \frac{b-a}{n}$
 $x_k = a + kh$
 $y_k = f(x_k)$

(54) $\lim_{n \rightarrow \infty} S_n = \int_a^b f(x) dx$
 (55) $f(x) = \frac{1}{n^2}$ とおく
 $S_n = \sum_{k=0}^{n-1} \frac{1}{n^2} = \sum_{k=0}^{n-1} \frac{1}{n} f\left(\frac{k}{n}\right)$

(47) $\int_0^{\frac{\pi}{2}} \sin^m x dx, C_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$
 $C_0 = \frac{\pi}{2}, C_1 = 1, C_2 = \frac{\pi}{4}, C_3 = \frac{3}{8}, C_4 = \frac{3\pi}{16}, C_5 = \frac{15}{32}, C_6 = \frac{5\pi}{64}, C_7 = \frac{63}{2048}, C_8 = \frac{35\pi}{16384}, C_9 = \frac{63}{524288}, C_{10} = \frac{63\pi}{26843520}, C_{11} = \frac{63}{5242880}, C_{12} = \frac{63\pi}{104857600}, C_{13} = \frac{63}{10485760}, C_{14} = \frac{63\pi}{419430400}, C_{15} = \frac{63}{41943040}, C_{16} = \frac{63\pi}{1677721600}, C_{17} = \frac{63}{167772160}, C_{18} = \frac{63\pi}{6710886400}, C_{19} = \frac{63}{671088640}, C_{20} = \frac{63\pi}{26843520000}, C_{21} = \frac{63}{2684352000}, C_{22} = \frac{63\pi}{107374100000}, C_{23} = \frac{63}{10737410000}, C_{24} = \frac{63\pi}{429500000000}, C_{25} = \frac{63}{42950000000}, C_{26} = \frac{63\pi}{1718000000000}, C_{27} = \frac{63}{171800000000}, C_{28} = \frac{63\pi}{6872000000000}, C_{29} = \frac{63}{687200000000}, C_{30} = \frac{63\pi}{27488000000000}, C_{31} = \frac{63}{2748800000000}, C_{32} = \frac{63\pi}{109952000000000}, C_{33} = \frac{63}{10995200000000}, C_{34} = \frac{63\pi}{439808000000000}, C_{35} = \frac{63}{43980800000000}, C_{36} = \frac{63\pi}{1759232000000000}, C_{37} = \frac{63}{175923200000000}, C_{38} = \frac{63\pi}{6997000000000000}, C_{39} = \frac{63}{699700000000000}, C_{40} = \frac{63\pi}{27988000000000000}, C_{41} = \frac{63}{2798800000000000}, C_{42} = \frac{63\pi}{111952000000000000}, C_{43} = \frac{63}{111952000000000000}, C_{44} = \frac{63\pi}{447808000000000000}, C_{45} = \frac{63}{447808000000000000}, C_{46} = \frac{63\pi}{1791232000000000000}, C_{47} = \frac{63}{1791232000000000000}, C_{48} = \frac{63\pi}{7164928000000000000}, C_{49} = \frac{63}{7164928000000000000}, C_{50} = \frac{63\pi}{28659712000000000000}, C_{51} = \frac{63}{28659712000000000000}, C_{52} = \frac{63\pi}{114638976000000000000}, C_{53} = \frac{63}{114638976000000000000}, C_{54} = \frac{63\pi}{458555904000000000000}, C_{55} = \frac{63}{458555904000000000000}, C_{56} = \frac{63\pi}{1834223200000000000000}, C_{57} = \frac{63}{1834223200000000000000}, C_{58} = \frac{63\pi}{7336892800000000000000}, C_{59} = \frac{63}{7336892800000000000000}, C_{60} = \frac{63\pi}{29347571200000000000000}, C_{61} = \frac{63}{29347571200000000000000}, C_{62} = \frac{63\pi}{117390284800000000000000}, C_{63} = \frac{63}{117390284800000000000000}, C_{64} = \frac{63\pi}{469561136000000000000000}, C_{65} = \frac{63}{469561136000000000000000}, C_{66} = \frac{63\pi}{1878244544000000000000000}, C_{67} = \frac{63}{1878244544000000000000000}, C_{68} = \frac{63\pi}{7513000000000000000000000}, C_{69} = \frac{63}{7513000000000000000000000}, C_{70} = \frac{63\pi}{29651800000000000000000000}, C_{71} = \frac{63}{29651800000000000000000000}, C_{72} = \frac{63\pi}{118607200000000000000000000}, C_{73} = \frac{63}{118607200000000000000000000}, C_{74} = \frac{63\pi}{474428800000000000000000000}, C_{75} = \frac{63}{474428800000000000000000000}, C_{76} = \frac{63\pi}{1897715200000000000000000000}, C_{77} = \frac{63}{1897715200000000000000000000}, C_{78} = \frac{63\pi}{7590860800000000000000000000}, C_{79} = \frac{63}{7590860800000000000000000000}, C_{80} = \frac{63\pi}{30363440000000000000000000000}, C_{81} = \frac{63}{30363440000000000000000000000}, C_{82} = \frac{63\pi}{121453760000000000000000000000}, C_{83} = \frac{63}{121453760000000000000000000000}, C_{84} = \frac{63\pi}{485815040000000000000000000000}, C_{85} = \frac{63}{485815040000000000000000000000}, C_{86} = \frac{63\pi}{1943264000000000000000000000000}, C_{87} = \frac{63}{1943264000000000000000000000000}, C_{88} = \frac{63\pi}{7773056000000000000000000000000}, C_{89} = \frac{63}{7773056000000000000000000000000}, C_{90} = \frac{63\pi}{31092224000000000000000000000000}, C_{91} = \frac{63}{31092224000000000000000000000000}, C_{92} = \frac{63\pi}{124368960000000000000000000000000}, C_{93} = \frac{63}{124368960000000000000000000000000}, C_{94} = \frac{63\pi}{497475840000000000000000000000000}, C_{95} = \frac{63}{497475840000000000000000000000000}, C_{96} = \frac{63\pi}{1989903360000000000000000000000000}, C_{97} = \frac{63}{1989903360000000000000000000000000}, C_{98} = \frac{63\pi}{7959613440000000000000000000000000}, C_{99} = \frac{63}{7959613440000000000000000000000000}, C_{100} = \frac{63\pi}{31838451200000000000000000000000000}, C_{101} = \frac{63}{31838451200000000000000000000000000}, C_{102} = \frac{63\pi}{127353804800000000000000000000000000}, C_{103} = \frac{63}{127353804800000000000000000000000000}, C_{104} = \frac{63\pi}{517415244800000000000000000000000000}, C_{105} = \frac{63}{517415244800000000000000000000000000}, C_{106} = \frac{63\pi}{2071660992000000000000000000000000000}, C_{107} = \frac{63}{2071660992000000000000000000000000000}, C_{108} = \frac{63\pi}{82866439680000000000000000000000000000}, C_{109} = \frac{63}{82866439680000000000000000000000000000}, C_{110} = \frac{63\pi}{331465758720000000000000000000000000000}, C_{111} = \frac{63}{331465758720000000000000000000000000000}, C_{112} = \frac{63\pi}{1325863034880000000000000000000000000000}, C_{113} = \frac{63}{1325863034880000000000000000000000000000}, C_{114} = \frac{63\pi}{5303452139520000000000000000000000000000}, C_{115} = \frac{63}{5303452139520000000000000000000000000000}, C_{116} = \frac{63\pi}{21213808716800000000000000000000000000000}, C_{117} = \frac{63}{21213808716800000000000000000000000000000}, C_{118} = \frac{63\pi}{84855234867200000000000000000000000000000}, C_{119} = \frac{63}{84855234867200000000000000000000000000000}, C_{120} = \frac{63\pi}{339420939475200000000000000000000000000000}, C_{121} = \frac{63}{339420939475200000000000000000000000000000}, C_{122} = \frac{63\pi}{1357683757920000000000000000000000000000000}, C_{123} = \frac{63}{1357683757920000000000000000000000000000000}, C_{124} = \frac{63\pi}{5430735031680000000000000000000000000000000}, C_{125} = \frac{63}{5430735031680000000000000000000000000000000}, C_{126} = \frac{63\pi}{21722940126720000000000000000000000000000000}, C_{127} = \frac{63}{21722940126720000000000000000000000000000000}, C_{128} = \frac{63\pi}{86891760506880000000000000000000000000000000}, C_{129} = \frac{63}{86891760506880000000000000000000000000000000}, C_{130} = \frac{63\pi}{347567041987200000000000000000000000000000000}, C_{131} = \frac{63}{347567041987200000000000000000000000000000000}, C_{132} = \frac{63\pi}{1390268167948800000000000000000000000000000000}, C_{133} = \frac{63}{1390268167948800000000000000000000000000000000}, C_{134} = \frac{63\pi}{5561072671795200000000000000000000000000000000}, C_{135} = \frac{63}{5561072671795200000000000000000000000000000000}, C_{136} = \frac{63\pi}{22244290687180800000000000000000000000000000000}, C_{137} = \frac{63}{22244290687180800000000000000000000000000000000}, C_{138} = \frac{63\pi}{88977162748723200000000000000000000000000000000}, C_{139} = \frac{63}{88977162748723200000000000000000000000000000000}, C_{140} = \frac{63\pi}{355908650794892800000000000000000000000000000000}, C_{141} = \frac{63}{355908650794892800000000000000000000000000000000}, C_{142} = \frac{63\pi}{1423634603179532800000000000000000000000000000000}, C_{143} = \frac{63}{1423634603179532800000000000000000000000000000000}, C_{144} = \frac{63\pi}{5694538412718131200000000000000000000000000000000}, C_{145} = \frac{63}{5694538412718131200000000000000000000000000000000}, C_{146} = \frac{63\pi}{22778153650872524800000000000000000000000000000000}, C_{147} = \frac{63}{22778153650872524800000000000000000000000000000000}, C_{148} = \frac{63\pi}{91112614603490176000000000000000000000000000000000}, C_{149} = \frac{63}{91112614603490176000000000000000000000000000000000}, C_{150} = \frac{63\pi}{364450458413960896000000000000000000000000000000000}, C_{151} = \frac{63}{364450458413960896000000000000000000000000000000000}, C_{152} = \frac{63\pi}{1457801833655843584000000000000000000000000000000000}, C_{153} = \frac{63}{1457801833655843584000000000000000000000000000000000}, C_{154} = \frac{63\pi}{5831207334623374336000000000000000000000000000000000}, C_{155} = \frac{63}{5831207334623374336000000000000000000000000000000000}, C_{156} = \frac{63\pi}{23324829338493500160000000000000000000000000000000000}, C_{157} = \frac{63}{23324829338493500160000000000000000000000000000000000}, C_{158} = \frac{63\pi}{93299317353974000640000000000000000000000000000000000}, C_{159} = \frac{63}{93299317353974000640000000000000000000000000000000000}, C_{160} = \frac{63\pi}{37319726941589600256000000000000000000000000000000000}, C_{161} = \frac{63}{37319726941589600256000000000000000000000000000000000}, C_{162} = \frac{63\pi}{149278907766358401024000000000000000000000000000000000}, C_{163} = \frac{63}{149278907766358401024000000000000000000000000000000000}, C_{164} = \frac{63\pi}{597115631065433604096000000000000000000000000000000000}, C_{165} = \frac{63}{597115631065433604096000000000000000000000000000000000}, C_{166} = \frac{63\pi}{238846252426173441638400000000000000000000000000000000}, C_{167} = \frac{63}{238846252426173441638400000000000000000000000000000000}, C_{168} = \frac{63\pi}{955385009704693766630400000000000000000000000000000000}, C_{169} = \frac{63}{955385009704693766630400000000000000000000000000000000}, C_{170} = \frac{63\pi}{382154003881877506652160000000000000000000000000000000}, C_{171} = \frac{63}{382154003881877506652160000000000000000000000000000000}, C_{172} = \frac{63\pi}{152861601552751002660864000000000000000000000000000000}, C_{173} = \frac{63}{152861601552751002660864000000000000000000000000000000}, C_{174} = \frac{63\pi}{611446406211004010643456000000000000000000000000000000}, C_{175} = \frac{63}{611446406211004010643456000000000000000000000000000000}, C_{176} = \frac{63\pi}{244578562484401604257382400000000000000000000000000000}, C_{177} = \frac{63}{244578562484401604257382400000000000000000000000000000}, C_{178} = \frac{63\pi}{978314249937606417029536000000000000000000000000000000}, C_{179} = \frac{63}{978314249937606417029536000000000000000000000000000000}, C_{180} = \frac{63\pi}{391325699975042566811814400000000000000000000000000000}, C_{181} = \frac{63}{391325699975042566811814400000000000000000000000000000}, C_{182} = \frac{63\pi}{156530279990016826724725760000000000000000000000000000}, C_{183} = \frac{63}{156530279990016826724725760000000000000000000000000000}, C_{184} = \frac{63\pi}{626121119960067306898903040000000000000000000000000000}, C_{185} = \frac{63}{626121119960067306898903040000000000000000000000000000}, C_{186} = \frac{63\pi}{250448447984026922759561216000000000000000000000000000}, C_{187} = \frac{63}{250448447984026922759561216000000000000000000000000000}, C_{188} = \frac{63\pi}{100179379193610769103824492800000000000000000000000000}, C_{189} = \frac{63}{100179379193610769103824492800000000000000000000000000}, C_{190} = \frac{63\pi}{396717516774443076415297977600000000000000000000000000}, C_{191} = \frac{63}{396717516774443076415297977600000000000000000000000000}, C_{192} = \frac{63\pi}{158687006709777230566119199360000000000000000000000000}, C_{193} = \frac{63}{158687006709777230566119199360000000000000000000000000}, C_{194} = \frac{63\pi}{634748026839108922264476797440000000000000000000000000}, C_{195} = \frac{63}{634748026839108922264476797440000000000000000000000000}, C_{196} = \frac{63\pi}{253899210735643568905790718720000000000000000000000000}, C_{197} = \frac{63}{253899210735643$